$$r \approx -t + \eta_D(1 + t)^{(4-h)/(2-h)},$$

it follows that the motion of the free boundary when $t > -1$ is accelerated.

The main result of the investigation is the following: The velocity of the free boundary remains constant in the time interval (t_0, t_1), where time t_0 corresponds to the beginning of the flow and t_1 coincides with the time t_f when the free boundary reaches the axis of the cylinder if the adiabatic index $\varkappa \leqslant 2$, and $t_1 < t_f$ if $\varkappa > 2$. In particular, when $\varkappa > 3$, the time t_1 coincides with the time t_0 if the gas was at rest before the flow began.

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EXTREMAL VALUES OF CYLINDER DRAG BEHIND A DISK IN SUPERSONIC FLOW

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UDC 533.601.1

The authors examine axisymmetric supersonic flow over a cylinder of diameter D, ahead of which is mounted a disk of diameter d < D on a thin connecting piece of length l. The flow separates on the disk, and near the body surface there is a flow circulation region, separated from the external flow by a mixing zone which spans a certain "dividing stream area" originating from the disk edge and incident on the end of the cylinder.

Taking account of the special features of the flow investigated, one can judge that the best procedure is to seek a solution of the problem based on a system of exact Navier–Stokes equations. However, with all the promise of this type of approach, even when adequately efficient numerical algorithms are available, a solution to the Navier–Stokes equations for a compressible fluid has been obtained as yet only for low and medium Reynolds numbers. An alternative approach is to construct an adequate mathematical model which would describe, as far as possible, the main characteristic features of the flow investigated.

As such an approximate model we choose a numerical model in which a result is obtained by applying the "large particle" numerical method [1, 2] to the equations describing the motion of an ideal gas — it reproduces the separated flow over the body in the process of establishing the solution corresponding to steady flow. The ideal fluid model has been used in a number of papers in investigating separated flows, including that at the front of a spiked body (cf. [3, 4]). Among the factors governing the fruitfulness of using this computational model the main one is evidently that it reproduces reliably the basic elements of the flow outside the circulation zone. The shape and dimensions of this zone are determined largely by the geometry of the components. Here we locate a large-scale unit vortex, separated from the walls and the outer flow by a comparatively thin viscous layer in which the transverse pressure gradient is small and which does not appreciably affect the pressure distribution on the body surface, at least above a certain Reynolds number (from Re $\geqslant$ 500, according to the data of [5]). One would expect that for the body of the composition considered here, with d < D, $l \sim$ D, the local Reynolds number for the flow in the circulation region will be large enough [5]. On the other hand, it is known that computational schemes for ideal gas flows similar to [1, 2], because of their inherent computational "dissipation" properties, give results with features characteristic of large Reynolds number flow. Finally, one should take into account the known idea that the base pressure depends only slightly on Re [6] in supersonic flow at large Reynolds number.

These considerations support the expectation that the numerical model will in the main correctly reflect the actual fluid flow in the entire computed region. The results then ob-

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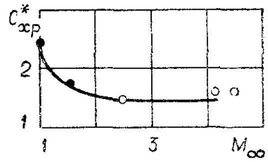

Fig. 1

tained for the pressure distribution on the body surface determine its profile pressure drag, and can be used subsequently in solving the problem of viscous flow and heat transfer directly in the wall regions. It is also clear that our understanding of the results of applying the computational model to the problem considered will improve as these results accumulate.

The main objective of the calculation is to generalize the data obtained in the numerical experiment directed at determination of the aerodynamic properties of a cylinder mounted behind a disk in supersonic flow, in order to evaluate the extremal characteristics of the configuration. The extremum of the pressure drag $C_{xp}(l, d, M_\infty)$ of the disk—cylinder combination, neglecting the contribution from the base pressure behind the cylinder, lies in the quadrant l, $d \geqslant 0$ (in defining C_{xp} here we take the characteristic area to be the cross section of the cylinder, and the characteristic dimension to be the cylinder diameter $D = 1$). For $d = 0$ and in the absence of the connecting piece (and for $d \leqslant 1$) C_{xp} is equal to the corresponding value for the cylinder without the disk C^*_{xp}. For large l it exceeds C^*_{xp} by the contribution of the separately washed disk, and for large d it increases as d^2 with increase of the disk diameter. In these conditions, because the drag $C_{xp} < C^*_{xp}$ at a certain point d, l, it follows that for $M_\infty = \text{const}$ there is a relative and an absolute extremum of the continuous function $C_{xp}(l, d)$.

The time-dependent method using the "large particle" scheme [1, 2] was employed to calculate steady supersonic flow, described by the set of parameters M_∞, $p_\infty = 1$, $\rho_\infty = 1$, $\varkappa = c_p/c_V = 1.4$, over the combination, at zero angle of attack. The criterion for settling of the solution in the computation was smallness of the increment of a characteristic gasdynamic parameter in the time layer.

Questions relating to formulation of the calculation have been explained in [7], which described in detail the main properties of the computing model, using an analysis of the pseudophysical process leading to the establishment of steady flow over the body, in a computation starting with certain nonphysical initial conditions. For example, the result was obtained that, independently of the choice of the initial state and the computing regime, the answer is single-valued in $\{l, d, M_\infty\}$, while the aerodynamic properties of the combination (in particular, the decrease of C_{xp} compared with C^*_{xp}) were formed in the calculation mainly by wave-type effects, due to interaction with the cylinder surface of the bow rarefaction wave propagating downstream from the disk, and with the reflected wave travelling towards the disk in the opposite direction.

Figure 1 shows the relation $C^*_{xp}(M_\infty)$ obtained in calculating the flow over the face of a cylinder, a typical blunt body of convex shape, adjoining the class of bodies considered ($l = 0$). Figure 1 also shows a number of experimental values of C^*_{xp} presented in [6] (open points), for Reynolds number defined with respect to D ($Re_\infty = 1.88 \cdot 10^6$), and our experimental data (closed points), obtained for $M_\infty = 0.98$, 1.55 ($Re_\infty = 1.1 \cdot 10^6$ and $6.2 \cdot 10^5$, respectively). These results show typical accuracy of the computation and can be used for a comparative analysis of the aerodynamic characteristics of the combinations presented below.

Figure 2a contains data on the comparative characteristics of the combination considered for $M_\infty = 1.77$, obtained by summing the results of the bulk calculations. Curves 1-16 show the $C_{xp}(d)$ relation for l, which is related to the number n by the formula $l = 0.125\,(n + 1)$. This information is presented more clearly in Fig. 2b. Here the isolines (of nominal pitch 0.05) show surfaces of level $C_{xp} = C_{xp}(l, d)$ for $M_\infty = 1.77$. It should be noted that the behavior of the surface at the edges of the region examined $l = 0.25$ and 2.125 exhibits the features described above. However, for $l = 2.125$ the value of C_{xp} is still far away from the sum of the contributions from the individually washed disk and cylinder (the absolute minimum in the relation $C_{xp}(l, d)$ located nearby still has a strong influence). The main feature of the relation is the presence in this region of a deep trough elongated along the l axis.

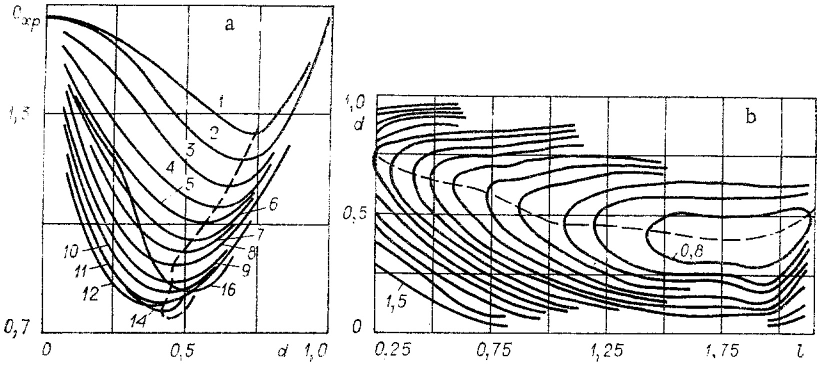

Fig. 2

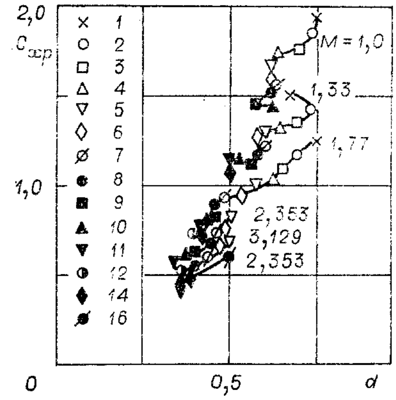

Fig. 3

By correlating results of this kind, obtained by computing the flow around the combination considered, located in a stream with various M_∞ values, one can construct the diagram shown in Fig. 3, where the solid lines indicate the geometric location of the relative minimum in the relation $C_{xp}(d)$ (this corresponds to the broken line in Fig. 2a). In Fig. 3 the parameter l for points n = 1-16 was calculated from the formula $l = 0.125 (n + 1)$.

In examining the results obtained one should have some understanding of their quality. In evaluating the "inherent" accuracy of the calculation, whose formulation is not retuned to the next variant, one would expect that it would fall off both as M_∞ becomes close to one and as l increases, due to the amplification in both cases of the influence of errors introduced by conditions of continuous extension at the boundaries of the computational region. Because of the limited computational accuracy one should not look for reliability in the details in the summed results like those of Fig. 2b, and to an even greater degree in Fig. 3. One can see certain circumstances where the deviation of the computing model from the actual process increases. It is evident that for $d \to 0$ the combination is seen by the flow as a spiked blunt body. The governing role of the disk as a generator of separation vanishes, and the nature of the separated flow is changed. Here the relative contribution of transport processes is increased. A description of this type of flow can be found in [8]. Similar circumstances arise for $l \gg 1$, because of the repeated reattachment of the flow to the connecting piece. The relative effect of the transport processes also increases with reduction of the intensity of flow circulation in the separated zone.

A review of results like those shown in Figs. 2 and 3 leads to a number of important conclusions.

In all the cases examined ($M_\infty < 4$, $0 < d < 1$, $0 < l < 2.5$) the computing model gives $C_{xp} < C^*_{xp}$. In the computing model it is easy to establish the existence of a minimum in the relation $C_{xp}(d)$ at each l in $0 < d < 1$.

In a number of cases we found a minimum in the relation $C_{xp}(d, l)$ (see Fig. 2). A relative minimum of $C_{xp}(d)$ was obtained for d, varying slightly with variation of l. These values decreased with increase of M_∞.

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GENERATION OF INTERNAL WAVES BY BOTTOM ROUGHNESS OF THE INTERFACE OF TWO FLUIDS FLOWING AT ANGLES TO EACH OTHER

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UDC 551.466.4

The simplest example of three-dimensional internal waves in a stream whose velocity varies with depth in both magnitude and direction is waves on the interfacial surface of two fluids of different densities flowing at an angle to one another. Investigation of the kinematic characteristics of the wave motion in such a fluid under the condition that the depth of the lower layer is infinite was performed in [1]. The asymptotic behavior of waves on the interfacial surface that occur during the flow around a body for the case of infinitely deep layers and of an obstacle on the bottom under the condition of infinite thickness of the upper layer was examined in [2]. The stability of waves occurring on the interfacial surface of two infinite streams flowing at an angle to each other was investigated in [3].

Let us consider the flow around an elevation described by the function f(x, z), by a stream infinite in the horizontal directions, in whose upper layer of thickness H_1 the fluid density is ρ_1, while it is $\rho_2 = \rho_1(1 + \varepsilon)(\varepsilon \geqslant 0)$ in the lower layer of thickness H_2. The velocity of the lower stream is U_2 and is along the x axis, while the velocity of the upper stream is U_1 and makes an angle α with the x axis. The x and z axes are on the unperturbed interfacial surface, the y axis is vertically upward, and the axis of symmetry of the obstacle passes through the origin.

Assuming the fluid motion within each layer to be irrotational, and the perturbations on the free surface and the interfacial surface to be small, we write the equations for the velocity potentials of the perturbed motion in each layer in the form

$$\Delta\varphi_1 = 0 \text{ for } 0 \leqslant y \leqslant H_1,\ \Delta\varphi_2 = 0 \text{ for } -H_2 \leqslant y < 0 \tag{1}$$

with boundary conditions on the free surface ($y = H_1$)

$$\partial\varphi_1/\partial y + L_1\zeta = 0,\ L_1\varphi_1 = g\zeta; \tag{2}$$

on the interfacial surface (y = 0)

$$\partial\varphi_1/\partial y + L_1\eta = 0,\ \partial\varphi_2/\partial y + L_2\eta = 0,\ \rho_2 L_2\varphi_2 - \rho_1 L_1\varphi_1 = g(\rho_2 - \rho_1)\eta; \tag{3}$$

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